



Estimation of Global Solar Radiation by Using Machine Learning Methods

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ABSTRACT

In this study, global solar radiation (GSR) was estimated based on 53 locations by using ELM, SVR, KNN, LR and NU-SVR methods. Methods were trained with a two-year data set and accuracy of the mentioned methods was tested with a one-year data set. The data set of each year was consisting of 12 months. Whereas the values of month, altitude, latitude, longitude, vapour pressure deficit and land surface temperature were used as input for developing models, GSR was obtained as output. Values of vapour pressure deficit and land surface temperature were taken from radiometry of NOAA-AVHRR satellite. Estimated solar radiation data were compared with actual data that were obtained from meteorological stations. According to statistical results, most successful method was NU-SVR method. The RMSE and MBE values of NU-SVR method were found to be 1,4972 MJ/m² and 0,2652 MJ/m², respectively. R value was 0,9728. Furthermore, worst prediction method was LR. For other methods, RMSE values were changing between 1,7746 MJ/m² and 2,4546 MJ/m². It can be seen from the statistical results that ELM, SVR, k-NN and NU-SVR methods can be used for estimation of GSR.

Keywords: Global Solar Radiation, NU-SVR

1 INTRODUCTION

The world has abundant solar energy resources. So that we need essentially to search sources of energy. Nowadays, climate change is increasing effects on human life. It shows us; why we must work more and more about this. One of them is GSR that is measured by a pyranometer but its installation is a very costly, takes long time and unusual way, especially in the developing countries. As mentioned, meteorological data based on many various factor and for each point, it cannot be measure easily. Cause of all these reason, it is modeled by some methods and it is estimated by this models. Estimation of daily GSR by the sunshine based models (Linear, Quadratic and Cubic) has been made and compared. The comparison results showed that the cubic model is equivalent to the quadratic model and the most constructive statistical results are given by the quadratic model [1]. Clearness index-beam transmittance numerical correlation was proposed using measured ambient temperature and relative humidity data to estimate hourly solar radiation [2]. Many researchers developed the correlations to predict the mean monthly diffuse solar radiation. However, most of these are based on regression analyses which are limited in accuracy and the number of parameters they can handle effectively [3,4]. The major drawback of the linear regression models is that they will underachieve when used to model nonlinear systems. The time series models like moving average, exponential smoothing and decomposition models are proposed for short term prediction of solar irradiance.



Many statistical and machine learning techniques have been used to forecast solar irradiance, including Autoregressive Moving Average (ARMA), Autoregressive Integrated Moving Average (ARIMA), Coupled Autoregressive and Dynamical System (CARDS), Artificial Neural Network (ANN), k-Nearest Neighbor (kNN), and Support Vector Regression (SVR), Extreme Learning Machines (ELM), linear regression (LR) [5]. In this study, the satellite based models and methods which are developed by combining a current statistical model with data obtained from satellite data is used. We profit by satellites. Because satellites; in very large areas, with short intervals, is able to scan in different phases of electromagnetic spectrum [6]. We used ELM, SVR, k-NN, LR and NU-SVR as estimating methods. The values of month, altitude, latitude, longitude, vapour pressure deficit and land surface temperature were used as input for developing models, GSR was taken as output. Land surface temperature and vapour pressure deficit have been estimated as monthly average by using NOAA-AVHRR satellite data in the thermal range. Generally, land surface temperature has been retrieved from two thermal infrared bands (channels 4 and 5 of AVHRR of NOAA) located at 11 µm and 12 µm by using split-window equations. The split-window algorithms are belonged to the difference in the brightness temperatures of thermal infrared bands. Furthermore, land surface temperature depends on the magnitude of the difference between the two grounds emissivity in the bands[8,7].

2 METHODOLOGY AND DATA SOURCES

In this study, GSR was predicted for 53 locations by using ELM, SVR, KNN, LR and NU-SVR methods. Methods were trained with a two-year data set and accuracy of the mentioned methods was tested with a one-year data set. The data set of each year was consisting of 12 months. Whereas the values of month, altitude, latitude, longitude, vapour pressure deficit and land surface temperature were used as input for developing models, GSR was obtained as output. Values of vapour pressure deficit and land surface temperature were taken from radiometry of NOAA-AVHRR satellite. Estimated solar radiation data were compared with actual data that were obtained from meteorological stations.

ELM has a very valuable methods of modeling tools, many of them with a proven track record in applications. We may consider here Support Vector Regression, one of the workhorses in nonlinear modeling, and, alternatively, we shall also work with two versions of SVR which depends only on a subset of the training data and NU-SVR, which was later developed where the epsilon penalty parameter was replaced by an alternative parameter, nu [0,1], which applies a slightly different penalty .In pattern recognition, the k-NN is a non-parametric method used for classification and regression. [9].LR measures the relationship between the categorical dependent variable and one or more independent variables by estimating probabilities using a logistic function, which is the cumulative logistic distribution.

2.1 Support Vector Regression

The main purpose of the SVR algorithm in time series forecasting [10] is to find a function that for each vector $\vec{x} \in R^n$ representing a time series within a dataset with N training time series sequences approximates its value ($i \ge 0 \le N$) as closely as possible. The result give us fundamentals for forecasting. When the input data are amenable to linear regression, SVR is defined by the equation (1);

$$y_i = \overrightarrow{x_i} \cdot \overrightarrow{w} + b \tag{1}$$

- \vec{w} is the weight vector, i.e., a linear combination of training patterns that supports the regression function.
- $\overrightarrow{x_l}$ is the input vector, e.g., the K_{T^*} time series training sample.
- y_i is the value for the input vector, e.g., the following K_{T^*} values to be predicted.
- b is the bias, i.e. $\frac{b}{\|\vec{w}\|}$ is the perpendicular distance from the origin of the vector space to the hyperplane that separates the data points in the vector space.



The objective of regression is to estimate the weight vector \vec{w} with the smallest possible length so as to avoid overfitting. To ease the regression task, a given margin of deviation ε is allowed with no penalty, and a given margin of deviation ε is allowed with increasing penalty. The minimal length of the weight vector \vec{w} is obtained by minimizing the loss function subject in equation (2) to the constraint in equation (3) or equation (4), for $\xi, \xi^*_i \ge 0$. The solution is given by constructing a Lagrange function from the loss function and the associated constraints, as shown in equation (5) where a_i and a^*_i are Lagrange multipliers [11]. The training vectors giving nonzero Lagrange multipliers are called support vectors and are used to construct the regression function. If the input data are not amenable to linear regression, then the vector data are mapped into a higher dimensional features space using a kernel function U Φ , such as the polynomial kernel: $\Phi(\vec{w}) \cdot \Phi(\vec{x_i}) = (1 + \vec{x_i} \cdot \vec{w})^3$.

$$\frac{1}{2} \|\vec{w}\|^2 + C \sum_{i=1}^n (\xi + \xi^*_i)$$
 (2)

$$y_i - (\vec{x} \cdot \vec{w} + b) \le \varepsilon + \xi_i$$
 (3)

$$y_{i} - (\vec{x} \cdot \vec{w} + b) \ge \varepsilon + \xi^{*}_{i} \tag{4}$$

$$y_i = \sum_{i=1}^n (a_i - a_i^*) (\overrightarrow{x_i} \cdot \overrightarrow{w}) + b, \text{ for } 0 \le i \le n$$

$$(5)$$

2.2 Extreme Learning Machine

The Extreme Learning Machine (ELM) is based on single hidden layer feed-forward network (SLFN) as Figure 1. Huang was the first to introduce the extreme learning machine algorithm. It is a new approach for feed forward networks that has a remarkable speed for mapping the relationship between input(s) and output(s). ELM creates a hidden layer without needing iterative steps and also computes the output weights analytically. There are no iterations in ELM, which makes ELM faster than the back propagation technique. However, there are some drawbacks of the ELM. The first issue is the neurons in the hidden layer have to be computed by using a trial-and-error procedure. The hidden layer needs more neurons because ELM generates random values chosen for the weighting matrix [12,13].

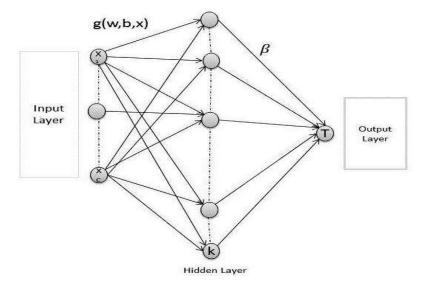


Figure 1: Extreme leaning machine process tree.



2.2.1 The ELM Algorithm

Suppose that we have training samples (x_i, t_i) where $x_i = [x_{i2}, x_{i2}, \dots, x_{in}]^T \in R_c$ and and $t_i = [t_{i2}, t_{i2}, \dots, t_{in}]^T \in R_m$. From these samples, an ELM model is trained with k hidden neurons and an activation function g(x). When ELM approximate training samples with zero error, we will obtain

$$\sum_{j=1}^{k} ||y_j - t_j|| = 0. \text{ In other words, } w_i, b_i \text{ and } x_i \text{ such that } [14]:$$

$$\sum_{i=1}^{k} \beta_i g(w_i, b_i, x_j) = t_i \tag{6}$$

The w_i is the input weight connected between input and hidden layers, b_i is the bias of the hidden layer, and is x_i the input sample. The equation (6) can be written as [14].

$$H\beta = T \tag{7}$$

Where

$$H = \begin{bmatrix} g(w_1, b_1, x_1) & \cdots & g(w_k, b_k, x_1) \\ \vdots & \ddots & \vdots \\ g(w_1, b_1, x_N) & \cdots & g(w_k, b_k, x_N) \end{bmatrix}_{Nxk}$$
(8)

$$\beta = \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_k \end{bmatrix} \tag{9}$$

$$T = \begin{bmatrix} T_1 \\ \vdots \\ T_n \end{bmatrix}$$
 (10)

H: is the hidden layer output

 β : is the output weight

T: is the target

 $\beta = H^{+}T$

H: is the Moore-Penrose generalized (pseudo-inverse) inverse. The orthogonal project method is used to calculate the Moore-Penrose generalized inverse of the matrix [14].

The ELM design involves four steps:

- 1. Dividing the data onto three subsets (training set, testing set, predicting set).
- 2. Generating the weight values randomly (w).
- 3. Computing the hidden layer output matrix (H).
- 4. Computing the output weight (β).

2.3 Linear Regression

Regression analysis is a statistical technique for investigating and modeling the relationship between variables as equation [15]. In fact, the regression analysis is the most widely used statistical technique. The simple linear regression model used is a model with a single independent variable *x* that has a relationship with a response variable *y* that is a straight line. This simple linear regression model is given by;

$$y = \beta_0 + x_1 \cdot \beta_1 + \varepsilon \tag{11}$$



Where the intercept β_0 and the slope β_1 are unknown constant and ε is a random error. The errors are assumed to have mean zero and unknown variance σ^2 . The parameters β_0 and β_1 are unknown and must be estimated using sample data. The simple linear regression equation is also called the *least squares* regression equation. It tells the criterion used to select the best fitting line, namely the sum of the *squares* of the residuals should be *least*. That is, the least squares regression equation is the line for which the sum of squared residuals $\sum_{i=1}^{n} (y_i - \widehat{y_i})^2$ is a minimum.

2.4 k-Nearest Neighbours

The k-nearest neighbors [16, 17] is a method for classifying objects based on closest training examples in the feature space. k-NN is a type of instance-based learning, or lazy learning where the function is only approximated locally and all computation is deferred until classification. It can also be used for regression. Unlike previous models, this tool does not use a learning base. The method consists in looking into the history of the series for the case the most resembling to the present case. By considering a series of observations X_t , to determine the next term X_{t+1} , we must find among anterior information, which minimize the quantity defined on equation.(12) (d is the quadratic error).

$$r_0 = \operatorname{argmin}(d(X_t, X_{t-r}) + d(X_{t-1}, X_{t-r-1}) + \dots \cdot d(X_{t-k}, X_{t-r-k})$$
(12)

In this study we have chosen a k equal to 10. After this argument of the minimum search, the prediction can be written as equation (13):

$$X_{t+1} = X_{t+r_0+1} (13)$$

2.5 Nu-support Vector Regression

This is a most commonly used versions of SWM. The original SVM formulations for Regression (SVR) used parameters C [0, inf) and epsilon [0, inf) to apply a penalty to the optimization for points which were not correctly predicted. An alternative version of both SVM regression was later developed where the epsilon penalty parameter was replaced by an alternative parameter, nu [0,1], which applies a slightly different penalty. The main motivation for the nu versions of SVM is that it has a more meaningful interpretation. This is because nu represents an upper bound on the fraction of training samples which are errors (badly predicted) and a lower bound on the fraction of samples which are support vectors. Some users feel nu is more intuitive to use than C or epsilon. Epsilon or nu are just different versions of the penalty parameter. [18]

The user must provide parameters (or parameter ranges) for SVM regression as:

• 'epsilon-SVR':

epsilon,C, (using linear kernel), or **epsilon**,C, **gamma** (using radial basis function kernel),

'nu-SVR':

nu, **C**, (using linear kernel), or

nu, C, gamma (using radial basis function kernel).

2.5.1 Sym Parameters

Cost: Cost [0 ->inf] represents the penalty associated with errors larger than epsilon. Increasing cost value causes closer fitting to the calibration/training data. Gamma: Kernel gamma parameter controls the shape of the separating hyperplane. Increasing gamma usually increases number of support vectors. Epsilon: In training the regression function there is no penalty associated with points which are predicted



within distance epsilon from the actual value. Decreasing epsilon forces closer fitting to the calibration/training data. Nu: Nu (0 -> 1] indicates a lower bound on the number of support vectors to use, given as a fraction of total calibration samples, and an upper bound on the fraction of training samples which are errors (poorly predicted).

3 EVALUATION OF THE ESTIMATION RESULTS

The choice of the relevant criteria allowing performance evaluation of the estimation methods is an important issue. Various statistical parameters can be used to measure the strength of the statistical relationship between the estimated values and the reference values. I assume that v_i , (i=1,n) is the set of n reference values and e_i , (i=1,n) is the set of the estimates. \bar{v} and \bar{e} are mean of reference and estimates values respectively. The bias, R, RMSE and MBE can be calculated by using standard deviations of reference (σ_v) and estimate (σ_e) values, mean of reference and estimates values, estimated values and the reference values. The bias which is the difference between the mean estimate \bar{e} and the mean reference value \bar{v} . The statistical criteria formula of the linear correlation coefficient R is the following,

$$r = \frac{\sum_{i=1}^{n} (v_i - \overline{v})(e_i - \overline{e})}{n\sigma_v \sigma_e}$$
(14)

Where R measures the proximity between estimate and reference. It is not sensitive to a bias [20]. The formula of the RMSE is;

$$RMSE = \left[\frac{1}{n} \sum_{i=1}^{n} (e_i - v_i)^2\right]^{\frac{1}{2}}$$
(15)

In statistics, RMSE is a frequently used measure of the differences between values predicted by a model or an estimator and the values actually observed from the thing being modeled or estimated [19]. MBE is;

$$MBE = \frac{1}{n} \sum_{i=1}^{n} \left[e_i - v_i \right] \tag{16}$$

When you compared actual value with the estimating result Mean Bias Error (MBE) is expected least value [21]. Ideal MBE is expected to approach zero. MBE can be negative or positive value. This is not important. MBE value is calculated with equation (16).

4 CONCLUSION

In this study, 2000, 2001 and 2002 years, GSR was estimated by using NOAA/AVHRR monthly mean land surface temperature and vapour pressure deficit. In addition to land surface temperature and vapor pressure deficit data, values of month, altitude, latitude, longitude were used to calculate GSR. In this estimating, GSR was estimated by depending on mentioned input parameters. Also ELM, SVR, KNN, LR and NU-SVR methods were used to train network. While input data of 2000 and 2001 year, were used to train network, data of 2002 year is used to test training network. Number is 53 for both intended training location and testing location. The obtained estimating results that depend on location, are evaluated with actual values statistically. As shown table 1.



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Iable I	Comparison	of methods	Statistically
I abic I	Comparison	or memous	statistically.

METOT	MBE(MJ/m ²)	RMSE(MJ/m ²)	R
ELM	-0,2640	1,7746	0,9612
SVR	-0,0043	1,9599	0,9515
k-NN	0,1450	2,4546	0,9250
LR	0,1588	5,2533	0,7243
Nu-SVR	-0,2652	1,4972	0,9728

When table 1 is examined, The highest correlation coefficient value have been found 0,9728. This result have been obtained by using Nu-SVR method. The lowest correlation coefficient value have been found 0,7243. The obtained result have been got by using LR method. On the other hand, MBE values of other methods is calculated. The lowest MBE value have been found 0,0043 MJ/m2. The obtained result have been got by using SVR method. The highest MBE value have been calculated 0,2652 MJ/m2. This result have been obtained by using Nu-SVR method. RMSE value also was calculated, which is important to compare statistically. According to the results, lowest RMSE value is 1,4972 MJ/m2, which have been obtained by using Nu-SVR method. So, It proved that this is the most successful method to estimate GSR. The highest RMSE value have been got by using LR method with 5,2533 MJ/m2 and also it proved that, this is the least successful method to estimate GSR. Other methods RMSE values are varying between 1,7746 MJ/m2 and 2,4546 MJ/m2. These results shows that, ELM, SVR, k-NN methods are enough for calculation.

Correlation coefficient (R) of locations, MBE, RMSE have been calculated by using the best method Nu-SVR.

Table 2 Statistical results of locations.

LOKASYON	MBE(MJ/m ²)	RMSE(MJ/m ²)	R
Amasya	-0,2008	1,2280	0,9812
Antakya	0,3185	0,6428	0,9946
Antalya	-2,0519	2,3409	0,9852
Balıkesir	-0,4160	1,4705	0,9713
Bingol	1,1040	1,9876	0,9772
Bitlis	-1,6446	1,9315	0,9875
Bursa	0,1060	1,2798	0,9749
Diyarbakır	-0,1068	1,0857	0,9877
Denizli	-0,7700	1,4644	0,9737
Edirne	-1,8345	2,1111	0,9887
Gaziantep	-0,2504	0,8017	0,9914
Gümüshane	-1,1129	1,5437	0,9883
Hakkarı	0,3520	1,6268	0,9727
Igdır	-0,4361	1,3683	0,9753
İstanbul	-0,1089	1,0919	0,9865
İzmir	-1,0915	1,8261	0,9800



Konya	-0,5819	1,5929	0,9720
Malatya	0,0596	0,8032	0,9921
Mersin	-0,4025	0,9471	0,9928
Mus	-0,5218	1,2138	0,9902
Nigde	-0,5780	1,2077	0,9868
Ordu	0,2149	1,2800	0,9761
Rize	0,4234	1,2807	0,9698
Siirt	-1,0340	1,5446	0,9818
Silifke	-0,0187	0,6101	0,9941
Sinop	1,7321	2,9830	0,9470
Sanlıurfa	-0,0859	1,5290	0,9738
Yalova	-0,2657	1,0187	0,9902
Yozgat	-0,2994	1,3885	0,9737
Zonguldak	0,2709	1,1093	0,9897
Adana	-0,5986	0,9866	0,9895
Adıyaman	0,0070	0,5777	0,9945
Agrı	-0,4485	1,3172	0,9751
Aksaray	0,1322	1,1816	0,9833
Aksehir	-0,3475	1,7290	0,9607
Ankara	-0,3934	1,0495	0,9881
Artvin	-0,2011	1,3190	0,9737
Aydın	-1,4823	2,1600	0,9716
Batman	2,0765	2,8554	0,9890
Bilecik	-0,5977	1,2318	0,9827
Birecik	-0,0764	0,6547	0,9949
Burdur	-0,5421	1,4751	0,9781
Canakkale	-1,1734	1,6766	0,9829
Corum	0,0200	1,1022	0,9831
Elazığ	-0,4757	1,0445	0,9913
Erzincan	0,0946	0,8903	0,9896
Isparta	1,7653	3,1973	0,8993
Kahramanmaras	0,0370	0,8804	0,9917
Karaman	-0,0706	1,2628	0,9824
Karatas	-1,1071	1,2870	0,9908
Kars	-0,3083	0,5266	0,9962
Kastamonu	-0,3836	1,4785	0,9526
Kayseri	-0,7520	1,3260	0,9863

When table 2 is examined, The highest correlation coefficient value have been found 0,9962. This result have been obtained from Kars location. The lowest correlation coefficient value have been calculated 0,8993. This result have been obtained from Isparta location. The lowest MBE value have been calculated 0,007 MJ/m2. This result have been obtained from Adiyaman location. The highest MBE value is 2,0765 MJ/m2which have been obtained from Batman location. The highest RMSE value have been



obtained 3,1973 MJ/m2. This result have been obtained from Isparta location. So the worst result have been obtained from Isparta location by using Nu-SVR method. The lowest RMSE value have been calculated as 0,5266 MJ/m2. The related value is belong to Kars location. So that, the best result have been obtained Kars location.

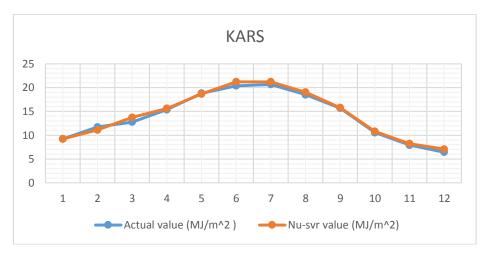


Figure 2: GSR variation of Kars location monthly.

Figure 2 shows 2002 year GSR variation of Kars location monthly. When we examine the variation, we can observed that estimated value and actual value is very close to each other. The first, fifth, ninth, tenth month's value are more compatible. It has been observed. We cannot say the same for the third and sixth month.

As a result, estimating was practiced by using 53 many locations. ELM, SVR, KNN, LR and NU-SVR methods were used to estimate GSR. Month, altitude, latitude, longitude, vapour pressure deficit and land surface temperature were used to train networks of methods, as input parameters. These obtained results have been evaluated statistically. The best result have been obtained by using Nu-SVR method. On the other hand, the worst result have been obtained by using LR method. Also the other methods gave compatible results. If researcher study estimating of GSR we may offer them to use ELM, SVR, KNN, and NU-SVR methods.

REFERENCES

- Yuehua L., Yingni J. and Xinxiao C. (2011). Evaluation of three models for calculating daily global solar radiation at Yushu, in Tibet Int. Conf. Cons. Electronic. Comm. and Network, 1252-1255.
- Al Riza D. F., Gilani S. I. H. and Aris M. S. (2011). Hourly solar radiation estimation using ambient temperature and relative humidity data Int. J. Environ. Sci. Dev., pp. 188-193.
- Park D. C., El-Sharkawi M. A. and Mark II R. J. (1991). Electric load forecasting using artificial neural network, IEEE Trans. Power Syst.99, pp. 442-449.
- Al-Alawi S. M. and Al-Hinai H. A. (1998). An ANN based approach for predicting global solar radiation in locations with no direct measurement instrumentation, Renew. Energy14, pp. 199-204.
- Mellit A. and Kalogirou S. A. (2008). Artificial intelligence techniques for photovoltaic applications: a review Prog. Energy Combust. Sci. 34 (5), pp. 574–632.



- [6] Şahin M., Yıldız Y., Şenkal O. and Peştamalci V. (2013). Estimation of the vapour pressure deficit using NOAA-AVHRR data J. of Rem. Sen. 34, pp. 2714-2729.
- [7] Şahin M. (2013). Weather Satellites and Remote Sensing LAP LAMBERT Ac. Pub. ISBN: 978-3-659-33947-9, pp. 1-68.
- [8] Becker F. (1987). The Impact of Spectral Emissivity on the Measurement of Land Surface Temperature International Journal of Remote Sensing 8, pp. 1509–1522.
- [9] Altman N. S. (1992). An introduction to kernel and nearest-neighbour nonparametric regression The American Statistician 46 (3), pp. 175–185.
- [10] Smola M., Smola A. J., Ratsch G, Schoölkopf B., Kohlmorgen J. and Vapnik V. (1997). Predicting time series with support vector machines Proceedings of ICANN'97Springer LNCS 1327, pp. 999–1004.
- [11] Smola A. and Schoölkopf B. (1998). A tutorial on support vector regression. NeuroCOLT Tech. Rep.TR 1998–030 (Royal Holloway College, London, U.K.)
- [12] Soh Y C., Huang G-B. and Lan Y. (2010). Constructive hidden nodes selection of extreme learning machine for regression Neurocomputing 73, pp. 3191-3199.
- [13] Qin A. K., Sugnthan P. N., Huang G.-B. and Zhu Q-Y. (2005). Evolutionary extreme learning machine The journal of the pattern recognition 38, pp. 1759-1763.
- [14] Wang D. H. and Lan Yand Huang G.-B. (2011). Extreme Learning Machines: a survey Int.J.Mach.Learn&Cyber2, pp. 107-122.
- [15] Montgomery D. C. and Peck E. A. (1992). Introduction to linear regression analysis 2nd ed. Wiley. New York.
- [16] Sharif M. and Burn D. H. (2006). Simulating climate change scenarios using an improved K-nearest neighbor model Journal of Hydrology 325 1-4, pp. 179-196.
- [17] Yakowitz S. (1987). Nearest neighbors method for time series analysis Journal of Time Series Analysis 8, pp. 235-247.
- [18] Liang H. and Xi-Long C. (2010). A nu-support vector regression based system for grid resource monitoring and prediction Acta Automatica Sinica 36, pp. 139–146.
- [19] Laurent H., Jobard I. and Toma A. (1998). Validation of satellite and ground-based estimates of precipitation over the Sahel Atmospheric Research, 47-48, 651-670.
- [20] Kendall M. A. and Stuart A. (1963). The advanced theory of statistics Griffin (Ed.), London, p. 1730.
- [21] Katiyar K., Kumar A., Pandey C. K. and Das B. (2010). Acomparative study of monthly mean daily clear sky radiation over India International Journal of Energy and Environment 1, pp. 177–182.